



Simulation of Vibration Resistance and the Ways for Its Improvement on Designing Multistage Constructions

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Authors' contributions

This work was carried out in collaboration between all authors. Authors ASS, AB, NV and AŠ designed the study, performed the analysis, wrote the protocol, and wrote the first draft of the manuscript and managed literature searches. Authors ASS and A. Bogorosh managed the analyses of the study and literature searches. All authors read and approved the final manuscript.

Article Information

DOI: 10.9734/BJAST/2015/17790

Editor(s):

(1) Jakub Kostecki, Department of Civil and Environmental Engineering, University of Zielona Góra, Poland.

Reviewers:

(1) Anonymous, CCS University, India.

(2) Lirong Wang, Stonybrook University, USA.

Complete Peer review History: <http://sciencedomain.org/review-history/9822>

Short Research Article

Received 26th March 2015
Accepted 19th May 2015
Published 18th June 2015

ABSTRACT

The paper considers the possibility of a theoretical definition of the natural frequencies of vibrations of thermoelectric coolers with a different number of stages and different configurations in the design phase. Determination of natural frequencies of vibrations in parallel was carried out for products, taking into account the damping produced by a number of structural elements as well - without taking into account the damping capacity of the solder layers at the ends of thermocouples. The analysis and comparison of the theoretical results were carried out.

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The results of the calculation determining the natural vibration frequencies of thermoelectric coolers (TEC) through a system of Lagrange equations of the 2nd kind were compared with the calculated values obtained by using the method of electrodynamic analogies.

Keywords: Resonance; the natural frequencies of vibration damping.

1. INTRODUCTION

To ensure the normal conditions of operation for electronic products and radio units included in electronic equipment and systems as well as upon striving to reduce the masses and sizes of the products, thermoelectric coolers (TEC) that's operation is based on the well-known Peltier principle are used [1]. The number of stages of TEC is chosen upon taking into account the purpose, specificity and parameters of the electronic products as well as their power and the heat emission [1,2,3].

As it is known, operation of any carrier is characterized by a certain frequency and a certain amplitude of the forced vibrations in the devices installed on it. Ensuring the normal conditions for operation of such devices, in particular TECs, and their mechanical consistency is bound with avoidance of appearance of resonance while the product is affected by forced vibrations of a certain spectrum of frequencies.

Thus, definition of the natural frequencies of vibrations of products in the phase of their design and development is an important engineering and scientific task. An accomplishment of the said task requires, in addition to experimental methods, also mathematical simulation that enables defining the needed parameters (such as sizes, shapes of structural elements, the relevant materials and so on) by calculations with the prescribed accuracy upon applying TECs of various constructions and complexity [4,5]. The general view of TEC that includes 3 stages is presented in Fig. 1.

In the paper, calculation of four-stage TEC construction that's parameters are specified below is considered.

2. MATERIALS AND METHODS

The number of thermoelectric elements in each stage of the products under research was: $n_1 = 12$, $n_2 = 24$, $n_3 = 54$, $n_4 = 124$ (in Fig. 1, a 3-stage cooler is shown as an example). The length of each thermoelectric element is $l = 1.4$ mm; the sizes of the cross-section: 0.7×0.7 mm. The

thermoelectric elements are made [6] of bismuth telluride Bi_2Te_3 with E-modulus $E = 60$ GPa.

The heat transfers (2) (Fig. 1) are made of 0.1 mm thick ceramic plates with interconnecting conductive copper paths plates on their surface. For fixing the thermoelectric elements to the conductive path on the lateral surface of the heat transfer, a solder is used; its thickness $\Delta = 1 \cdot 10^{-1}$ m and the accepted E-modulus $E_{\Pi} = 12$ GPa.

The masses of heat transfers at the above-mentioned number of thermoelectric elements were following: $m_1 = 10.92896 \cdot 10^{-5}$ kg; $m_2 = 4.55961 \cdot 10^{-5}$ kg; $m_3 = 1.95753 \cdot 10^{-5}$ kg; $m_4 = 1.31891 \cdot 10^{-5}$ kg. In spite of sufficiently small sizes and mass, such products enable achieving the temperature differential up to 20°C on each stage, thus ensuring considerable cooling of the operating electronic devices and products. TEC installation in mobile carriers (such as ground-based, underwater, aerial, and cosmic) enables to reduce considerably the total weight of the commutations-electronics equipment and the sizes of the whole installation.

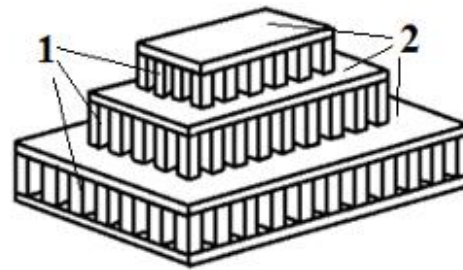


Fig. 1. The general view of the 3 – stage construction: 1 – thermoelectric elements; 2 – heat transfers

The natural frequencies of vibrations of TEC (of multistage or cordwood structure) were found upon applying the developed mathematical model based on using a system of Lagrange equations of the 2nd kind [7,8]:

$$\frac{d}{dt} \left(\frac{\partial T}{\partial \dot{x}_i} \right) - \frac{\partial T}{\partial x_i} = - \frac{\partial \Phi}{\partial x_i} - \frac{\partial \Pi}{\partial x_i} \quad (1)$$

where: T – kinetic energy of the system; i – the generalized coordinate; x_i – the shift of the i -th stage, $i = 1, 2, 3, 4$; Φ – dissipative function; t – time; Π – potential energy of the system.
Kinetic energy of the system:

$$T = \sum_{i=1}^4 T_i \quad (2)$$

Potential energy of the system:

$$\Pi = \sum_{i=1}^4 \Pi_i \quad (3)$$

Dissipative function is proportional to the speed of motion of masses of the system:

$$\Phi = \sum_{i=1}^4 \Phi_i \quad (4)$$

$$\Phi_i = \frac{1}{2} \beta_i \dot{x}_i^2 \quad (5)$$

where β_i – the damping coefficient.

The paper considers the structure of four-stage TEC that schematically may be presented in Fig. 2.

In the paper, the specific 4-stage TEC, where $i = 1, 2, 3, 4$, is discussed upon. Here m_i are assumed the masses of the relevant heat transfers and c_i – the stiffness of the thermoelectric elements of each stage, respectively [4].

The values of E-modulus of the solder layers at the ends of thermocouples are considerably lower, as compared to the analogous parameter of the material of the thermocouples themselves, i.e. its stiffness is lower. So, the solder layers at the ends of thermocouples may be considered peculiar dampers that reduce the values of the natural frequencies of the product.

The dissipative function is known to be non-linear [7]. Taking the said non-linearity into account in complex engineering systems is bound with considerable difficulties, including mathematical ones. In many cases, its taking into account enables to accomplish considerable refinement of the sought-for values of parameters upon taking into consideration the specificity of products and the conditions of their use.

Because of small sizes and their specificity, it was supposed in the work that the dissipative forces might be ignored in the design phase. The assumption was verified by calculations. The discrepancy between values of the results was 5%.

The analysis of the accepted approximate mathematical model for 2-stage, 3-stage and 4-stage TECs (upon applying the specifically developed software) enabled establishing the values of natural frequencies both for the discussed cases of constructions of the products and for any number of thermoelectric elements in a stage that is predetermined by the purpose of a specific TEC.

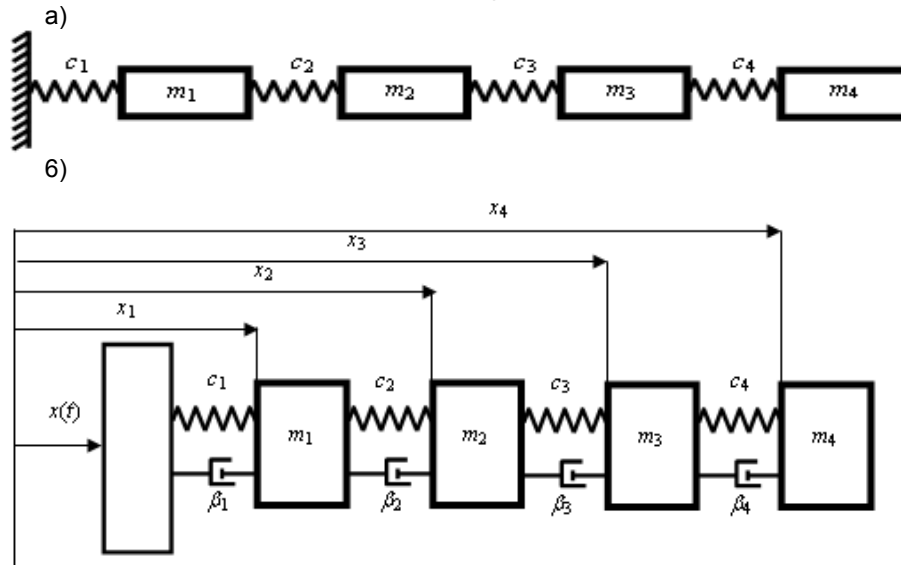


Fig. 2. The design model of four-stage TEC; a) when damping is ignored; b) when damping is taken into account

3. RESULTS AND DISCUSSION

The impact of the thickness of the solder layers on the values of natural frequencies, including the case of absence of damping, was analyzed. Cases of various combinations of thermoelectric elements in each stage as well as the particular case when stiffness of all stages of the product is the same were considered. The carried out research enabled establishing the dependence of changes of the values of natural frequencies on presence of dampers, their physical and mechanical properties and parameters as well as on parameters of stiffness of each stage.

The obtained results enable forecasting of a possibility of appearance of resonance in the systems as early as in the phase of design and development of future products and taking measures for “drifting” the natural frequencies of the products aside from the values of resonance frequencies. It is particularly important in development of responsible and expensive systems operating on movable carriers with certain values of forced vibrations.

To verify the adequacy of the results obtained by calculations, the natural frequencies of the vibrations of the above-discussed TECs were found upon applying the independent approach – the method of electrodynamic analogies (EDA) known in engineering mechanics. For this

purpose, the mechanical system under discussion is replaced with relevant electric analogies.

The electric scheme of the four-stage thermoelectric cooler is presented in Fig. 3.

For convenience of analysis and comparison, the results obtained upon applying two independent calculation methods are provided in Tables 1–4.

The analysis of the data provided in the Tables 1 and 2 attests that scattering of the values of natural frequencies depends on the number of elements in a stage and varies in the limits 1.94% - 4.51% (when damping is taken into account). If damping is not taken into account, the scattering of the values varies in the limits 5.27% - 5.19%. If the number of elements in a stage is reduced, the scattering of the results increases in both cases. If damping is taken into account, the obtained values of the natural frequencies, as in the earlier discussed cases, are by two orders of magnitude less (according to the data provided in Tables 1 and 2) that may explained by a considerable impact of the dampers on the values of natural frequencies of the products. So, it becomes possible to “adjust” the values of natural frequencies by changing them towards the values needed for the design engineer.

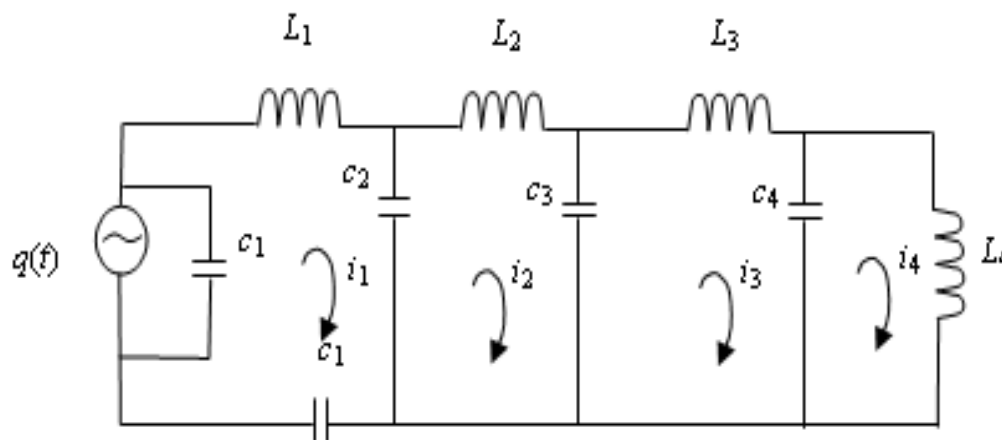


Fig. 3. The electric schematization of the four-stage TEC

Table 1. The values of natural frequencies of 3-stage product (Hz) at $n_1 = 124$, $n_2 = 54$, $n_3 = 24$ when damping is taken into account

The number of thermoelectric elements in a stage	124	54	24
Lagrange differential equations of the 2 nd kind	$2.177 \cdot 10^4$	$4.027 \cdot 10^4$	$5.50 \cdot 10^4$
EDA	$2.22 \cdot 10^4$	$4.234 \cdot 10^4$	$5.76 \cdot 10^4$

Table 2. The values of natural frequencies of 3-stage product (Hz) at $n_1 = 124$, $n_2 = 54$, $n_3 = 24$ when damping is not taken into account

The number of thermoelectric elements in a stage	124	54	24
Lagrange differential equations of the 2 nd kind	$2.949 \cdot 10^6$	$5.455 \cdot 10^6$	$7.45 \cdot 10^6$
EDA	$3.113 \cdot 10^6$	$5.878 \cdot 10^6$	$7.821 \cdot 10^6$

Table 3. The values of natural frequencies (Hz) of a 4-stage when damping is taken into account

The number of thermoelectric elements in a stage	124	54	24	12
Lagrange differential equations of the 2 nd kind	$1.797 \cdot 10^4$	$3.187 \cdot 10^4$	$4.682 \cdot 10^4$	$5.741 \cdot 10^4$
EDA	$1.842 \cdot 10^4$	$3.324 \cdot 10^4$	$4.791 \cdot 10^4$	$5.913 \cdot 10^4$

Table 4. The values of natural frequencies (Hz) of a 4-stage system when damping is not taken into account

The number of thermoelectric elements in a stage	124	54	24	12
Lagrange differential equations of the 2 nd kind	$2.434 \cdot 10^6$	$4.318 \cdot 10^6$	$6.344 \cdot 10^6$	$7.774 \cdot 10^6$
EDA	$2.033 \cdot 10^6$	$4.93616 \cdot 10^6$	$6.872 \cdot 10^6$	$8.234 \cdot 10^6$

The analysis of the data presented in the Tables 3 and 4 above attests that scattering of the values of natural frequencies of vibrations obtained in two independent ways varies in the limits 2.44% - 2.91%, if damping is taken into account; if damping is ignored in the method of calculation, the scattering varies in the limits 6.49% - 5.58%.

4. CONCLUSION

The results obtained in two independent ways showed the convergence of 3-8%, thus confirming the duly quality and adequacy of the mathematical models used in the work.

Analysis and comparison of the results of calculation obtained upon applying two independent methods enable assessing the possible values of natural frequencies of vibrations of specific products intended for practical use and those planned for manufacturing and use as early as in the design phase.

The developed mathematical models based on Lagrange differential equations of the 2nd kind may be successfully used as algorithms and programs for personal computers and enable establishing the natural frequencies of vibrations of TECs of various configurations and various embodiments as well as establishing the most acceptable version for each type of a device carrier. Permanent development and appearance of new engineering materials usable in electronics enables effective choosing of

combinations required for each particular product, thus ensuring the duly quality and performance capacity of devices and complex engineering systems.

COMPETING INTERESTS

Authors have declared that no competing interests exist.

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